

Application of the Finite Difference Method (FDM) and the Finite Element Method (FEM) to Solve a Thermal Problem

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Problem

FDM:

consists in getting approximate solutions / approximating derivatives by finite differences.

↳ **Matlab®**

Objectives:

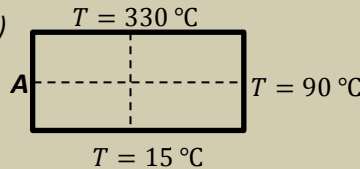
FEM:

consists in finding approximate solutions to boundary value problems for differential equations.

↳ **Abaqus®**

a) Temperature distribution; b) Heat flux distribution.

Inox Steel Plate 18x6 ($\lambda=16 \text{ W/m } ^\circ\text{C}$)



Different boundary conditions:

i) $T_A = 175 \text{ } ^\circ\text{C}$ ii) $\dot{q}_{A,in} = 5 \text{ W/m}^2$

Procedure

Table 1: procedure to get different discretizations of the mesh

Size of elements [m]	No. of Nodes	No. horizontal x vertical elements
3	5	2x6
1,5	33	4x12
1	85	6x18
0,75	161	8x24
0,5	385	12x36
0,3	1121	20x60
0,15	4641	40x120

Temperature

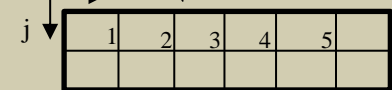
$$T_{i+1,j} + T_{i,j+1} - 4T_{i,j} + T_{i-1,j} + T_{i,j-1} = 0$$

$$\dot{q}_x = -\frac{\lambda}{2\Delta x} (-3T_{i,j} + 4T_{i+1,j} - T_{i+2,j})$$

Heat Flux

$$\dot{q}_x = -\lambda \left(\frac{\partial T}{\partial x} \right) \quad \dot{q}_y = -\lambda \left(\frac{\partial T}{\partial y} \right)$$

$$\dot{q} = \sqrt{\dot{q}_x^2 + \dot{q}_y^2}$$



Results

385 nodes (12 x 36 elements)

FDM

i)

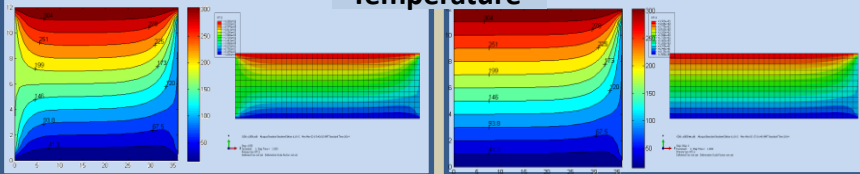
FEM

FDM

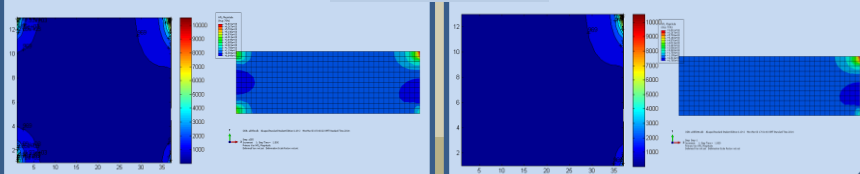
ii)

FEM

Temperature

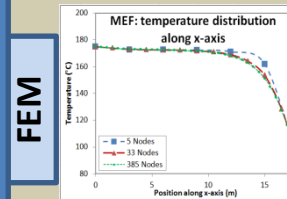
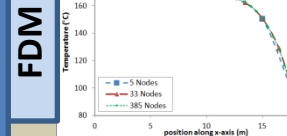


Heat Flux



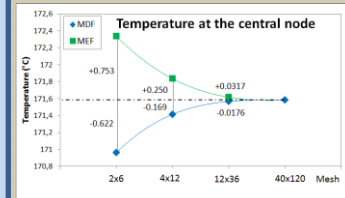
Results Analysis

Temperature (along X axis)

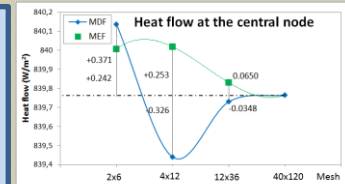


Errors of the FDM and FEM methods

Central Node



Heat Flux



Conclusions: FDM and FEM are powerful methods which allow us to know the temperature and heat flux distribution in a plate. The results are better if a more discretized mesh is used in the attempt.