Application of the Finite Difference Method (FDM) and the Finite Element Method (FEM) to Solve a Thermal Problem

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Problem

FDM: consists in getting approximate solutions / approximating derivatives by finite differences.

↳ **Matlab®**

FEM: consists in finding approximate solutions to boundary value problems for differential equations.

↳ **Abaqus®**

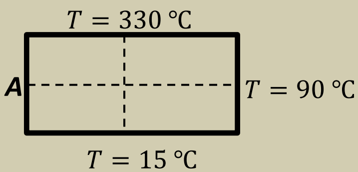
Objectives:

a) Temperature distribution; b) Heat flux distribution.

Inox Steel Plate 18x6 (λ=16 W/m ºC)

Different boundary conditions:

i) T_A **= 175 ºC** **ii)** $\dot{q}_{A,in}$ **=5 W/m²**



Procedure

Table 1: procedure to get different discretizations of the mesh

Size of elements [m]	No. of Nodes	No. horizontal x vertical elements
3	5	2x6
1,5	33	4x12
1	85	6x18
0,75	161	8x24
0,5	385	12x36
0,3	1121	20x60
0,15	4641	40x120

Temperature

i)

$$\underbrace{T_{i+1,j}}_{\rightarrow} + \underbrace{T_{i,j+1}}_{\uparrow} - 4T_{i,j} + \underbrace{T_{i-1,j}}_{\leftarrow} + \underbrace{T_{i,j-1}}_{\downarrow} = 0$$

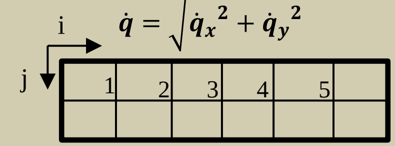
ii)

$$\dot{q}_x = -\frac{\lambda}{2\Delta x}\left(-3T_{i,j} + 4T_{i+1,j} - T_{i+2,j}\right)$$

Heat Flux

$$\dot{q}_x = -\lambda\left(\frac{\partial T}{\partial x}\right) \qquad \dot{q}_y = -\lambda\left(\frac{\partial T}{\partial y}\right)$$

$$\dot{q} = \sqrt{\dot{q}_x{}^2 + \dot{q}_y{}^2}$$

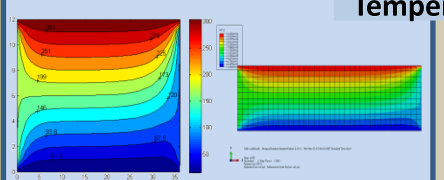


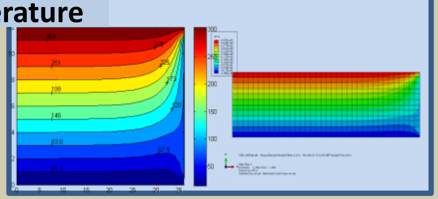
Results

385 nodes (12 x 36 elements)

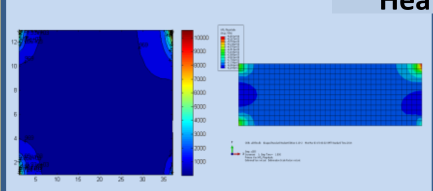
FDM	i)	FEM	FDM	ii)	FEM

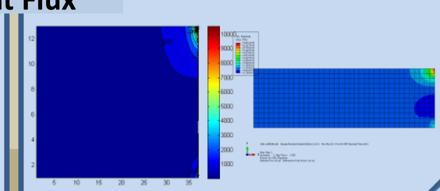
Temperature





Heat Flux

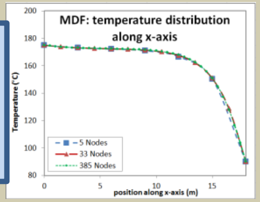




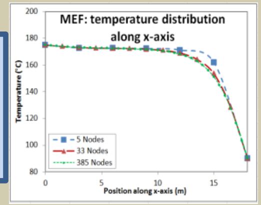
Results Analysis

Temperature (along X axis)

FDM



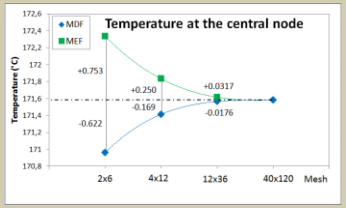
FEM



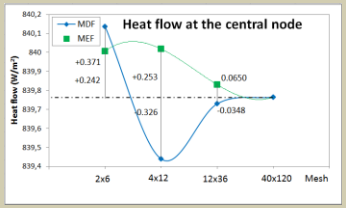
Errors of the FDM and FEM methods

Central Node

Temperature



Heat Flux



Conclusions: FDM and FEM are powerful methods which allow us to know the temperature and heat flux distribution in a plate. The results are better if a more discretized mesh is used in the attempt.

Course: Simulação de Processos Tecnológicos

Professor: Eng. Abel Dias dos Santos